VAQEM: A Variational Approach to Quantum Error Mitigation

Gokul Subramanian Ravi*
Postdoc, University of Chicago / EPIQC

**UC**Chicago: Kaitlin Smith, Fred Chong
**IBM**: Nate Earnest-Noble, Ali Javadi-Abhari
**Super.tech**: Pranav Gokhale
**Unitary Fund**: Andrea Mari

CCF-1730082/1730449, NSF Phy-1818914, DE-SC0020289, DE-SC0020331, NSF OMA-2016136, Q-NEXT DOE NQI Center, CIFellows (NSF 2030859) and IBM/CQE.
Summary: A variational approach to quantum error mitigation

• **Background:** VQAs are considered suitable to the NISQ era, but machine fidelity is still too low for real world applicability.

• **Goal:** Apply error mitigation in an optimal manner to VQAs for max fidelity – but this is challenging as device and circuit complexity increase.

• **Proposal:**
  • Integrate EM techniques into VQA’s framework of iterative parameter tuning: enabling a feedback-based approach towards optimal EM for the application / device.
  • Targets two idle-time EM methods: insertion of dynamical decoupling sequences and single-qubit gate scheduling.

• **Result:** Improves the quality of the VQA measured objective by 3x on average.
Variational Quantum Algorithms

Classical Optimizer: Tuning gate rotations

Objective Function (Minimize)

Ansatz circuits w/ different measurements

Gate angle parameters

Minimize:

\[ H = [ \text{XIII} + \text{IXII} + \text{IIIX} + \text{IIIIX} \\
+ \text{IIIXI} + \text{IIIXX} + \text{ZZIII} + \text{IZZIII} \\
+ \text{IIIZXI} + \text{IIIZZX} + \text{IIIZZX} + \text{ZIIIIZ} ] \]

Objective Function

(Minimize)

Gate angle parameters

Potential energy curve of a molecule

Variational principle: the energy of any trial wave-function is greater than or equal to the exact ground state energy

\[
\frac{\langle \Psi(\vec{\theta}) | H | \Psi(\vec{\theta}) \rangle}{\langle \Psi(\vec{\theta}) | \Psi(\vec{\theta}) \rangle} \geq E_G
\]

https://qiskit.org/learn/intro-qc-qh/
VQA Fidelity in the NISQ era

Energy ($E$) = $Tr[H_{\rho}]$
= $\langle \phi | H | \phi \rangle$ (pure state)
Impact of Error Mitigation

Energy (E) = Tr[Hρ]
            = ⟨ϕ|H|ϕ⟩ (pure state)

Different EM configurations

Tr[Hρ] for typical, noisy VQE
Tr[Hρ] for VAQEM
Noise-free optimization surface

Noisy outcomes
Error-mitigated outcomes
Region beyond lower bound

Parameter Space
Compiling for machines with limited connectivity leads to increased depth and long critical paths.

As application sizes increase, path lengths become longer and more diverse leading to more slack.

Decoherence in idle window.

Mapped to IBM Q Manhattan.

Extrapolated to anticipate near-term machines.
Idle Window Signal Refocusing: 1Q gate scheduling

Spin Echo Correction: Details in the paper!

optimal position?
Idle Window Signal Refocusing: Dynamic decoupling

Spin Echo Correction: Details in the paper!

optimal gate types / number / spacing?
Optimizing EM: practical challenges

1) Imperfect knowledge of stimuli and their effects makes theory driven EM heuristics less effective.
Optimizing EM: practical challenges

2) Micro-analyzing stimuli effects for every EM instance is not scalable.
Optimizing EM: practical challenges

3) Stimuli-agnostic outcome driven approaches are not always possible since outcomes are often unknown and usually not of highest probability.

* Ensemble of Diverse Mappings MICRO2019
VAQEM: Tuning EM features in the VQA setting

Design details in the paper!
VAQEM: Tuning EM features in the VQA setting

Minimize Objective (H)

Energy (E) = \( \text{Tr}[\mathcal{H}_p] \)

Only *quantum* EM – details in the paper!
VQE benefits from VAQEM I

<table>
<thead>
<tr>
<th>Bench</th>
<th>6q/f/2r</th>
<th>6q/c/2r</th>
<th>4q/c/6r</th>
<th>4q/f/6r</th>
<th>6q/c/4r</th>
<th>Li+</th>
<th>H2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>54</td>
<td>31</td>
<td>57</td>
<td>101</td>
<td>55</td>
<td>90</td>
<td>61</td>
</tr>
<tr>
<td># Win</td>
<td>42</td>
<td>24</td>
<td>22</td>
<td>34</td>
<td>30</td>
<td>45</td>
<td>26</td>
</tr>
</tbody>
</table>

VQE Energy Rel. Baseline (Neg)
More in the paper:

1. Ideal Flow vs Feasible Flow
2. Graph showing fraction of maximum against idle window #
3. Graph showing VQE Energy (Rel. Optimal) for various quantum states
4. Graph showing tuning angles and time (minutes)
5. Graph showing objective function values
6. Graphs showing BMRO Cos wave model sim and BMRO Cos feedback machine sim
Conclusion: A variational approach to quantum error mitigation

Future Directions:
- Variationally tune more features of current EM techniques
- Integrate more EM techniques into the VAQEM framework
- Explore tunable optimizations outside of error mitigation
Thank you!

gravi@uchicago.edu

VAQEM: arXiv:2112.05821
TimeStitch: arXiv:2105.01760
Backup
VQE benefits from VAQEM I

<table>
<thead>
<tr>
<th>Bench</th>
<th>6q/f/2r</th>
<th>6q/c/2r</th>
<th>4q/c/6r</th>
<th>4q/f/6r</th>
<th>6q/c/4r</th>
<th>Li+</th>
<th>H2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>54</td>
<td>31</td>
<td>57</td>
<td>101</td>
<td>55</td>
<td>90</td>
<td>61</td>
</tr>
<tr>
<td># Win</td>
<td>42</td>
<td>24</td>
<td>22</td>
<td>34</td>
<td>30</td>
<td>45</td>
<td>26</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VAQEM: GS</th>
<th>XY</th>
<th>VAQEM: XY</th>
<th>XX</th>
<th>VAQEM: XX</th>
<th>VAQEM: GS+XY</th>
</tr>
</thead>
<tbody>
<tr>
<td>HW_TFIM_6q_f_2r</td>
<td>1.2</td>
<td>2.8</td>
<td>2.9</td>
<td>2.1</td>
<td>1.6</td>
</tr>
<tr>
<td>HW_TFIM_6q_c_2r</td>
<td>1.3</td>
<td>1.2</td>
<td>1.1</td>
<td>1.4</td>
<td>2.1</td>
</tr>
<tr>
<td>HW_TFIM_4q_c_6r</td>
<td>1.7</td>
<td>1.7</td>
<td>1.3</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>HW_TFIM_4q_f_6r</td>
<td>1.6</td>
<td>1.6</td>
<td>1.3</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>HW_TFIM_6q_c_4r</td>
<td>11.6</td>
<td>11.6</td>
<td>11.6</td>
<td>11.6</td>
<td>11.6</td>
</tr>
<tr>
<td>HW_Li+</td>
<td>7.0</td>
<td>7.0</td>
<td>7.0</td>
<td>7.0</td>
<td>7.0</td>
</tr>
<tr>
<td>UCCSD_H2</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Geo Mean</td>
<td>3.02</td>
<td>3.02</td>
<td>3.02</td>
<td>3.02</td>
<td>3.02</td>
</tr>
</tbody>
</table>
EM inspired by spin echo correction
Ideal Flow

Qiskit Runtime

- Circuit w/ angle + EM parameters
- Optimal Classical Tuner
- Best suited Quantum Machine

Feasible Flow

Simulation

- Circuit w/ angle parameters
- Optimal Classical Tuner
- Noise-free Computation Model

Qiskit Runtime

- Circuit w/ angle parameters
- Available Classical Tuner
- Available Quantum Machine

In-house independent Window EM Tuner

- Opt. angle circuit w/ EM parameters
- Per-window optimal EM search
- Available Quantum Machine
Tunable Error Mitigation Scope

Energy ($E$) = $Tr[H_p]$
= $\langle \phi | H | \phi \rangle$ (pure state)

$\frac{\langle \Psi(\tilde{\theta}) | H | \Psi(\tilde{\theta}) \rangle}{\langle \Psi(\tilde{\theta}) | \Psi(\tilde{\theta}) \rangle} \geq E_G$

Classical post processing EM

Noisy outcomes

Error-mitigated outcomes

Region beyond lower bound

Proof in the paper!

* Quantum EM techniques *
VAQEM Tuning Overheads

![Bar chart showing tuning angles and other metrics for different tasks and problems.](image-url)
<table>
<thead>
<tr>
<th>Bench</th>
<th>6q/f/2r</th>
<th>6q/c/2r</th>
<th>4q/c/6r</th>
<th>4q/f/6r</th>
<th>6q/c/4r</th>
<th>Li+</th>
<th>H2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth</td>
<td>54</td>
<td>31</td>
<td>57</td>
<td>101</td>
<td>55</td>
<td>90</td>
<td>61</td>
</tr>
<tr>
<td># Win</td>
<td>42</td>
<td>24</td>
<td>22</td>
<td>34</td>
<td>30</td>
<td>45</td>
<td>26</td>
</tr>
</tbody>
</table>
VQA Fidelity in the NISQ era

* Classical Optimizers for Noisy Intermediate-Scale Quantum Devices QCE 2020